

TECHNICAL APPLICATION BULLETIN

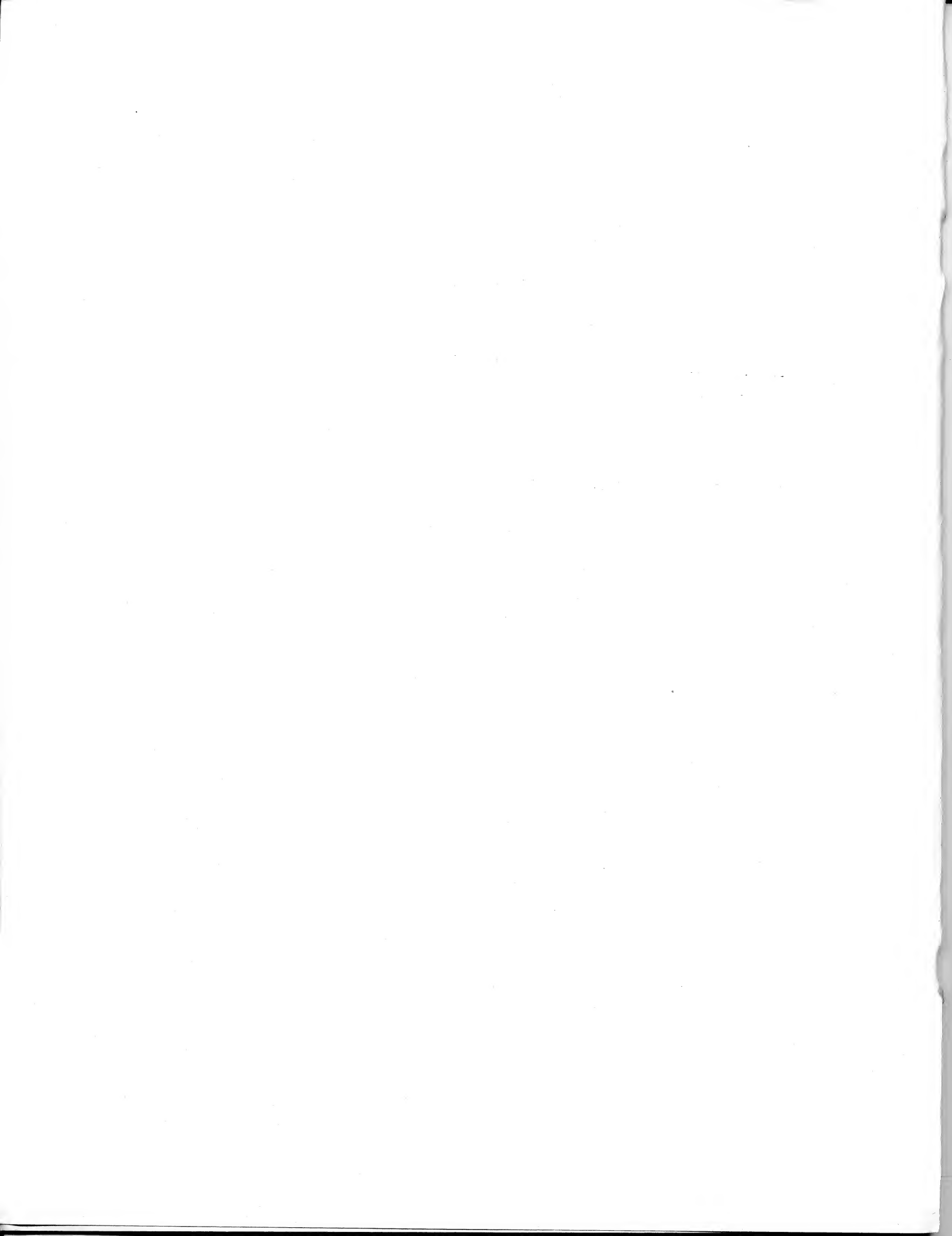
No. 4

COMPUTATION OF AUTO- AND CROSSCORRELATION FUNCTIONS

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COMPUTATION OF AUTO- AND CROSSCORRELATION FUNCTIONS

INTRODUCTION

Data representing the time history of physical and biological phenomena can be generally classified as either deterministic or random. A purely deterministic function can be completely described in terms of an explicit mathematical expression which indicates its value at any instant of time. If the data cannot be so described, it is regarded as random. Since each time record is unique, random phenomena can only be described in terms of their statistical properties.

Correlation is one of the statistical methods most commonly applied to the analysis of random data. This analytical technique provides a means for extracting information and describing random data in the time domain, as opposed to either the frequency or amplitude domain.

Normally, correlation is used to detect hidden periodicities within a signal or to provide a quantitative measure of the interdependency between two signals. The problems most often solved are those which require isolation of a known or unknown periodicity from random activity, or distinguishing between two sources of random activity. Auto-correlation measures the similarity of a signal to a time-delayed version of itself, while crosscorrelation measures the degree of similarity of one source or input to a time-delayed second source.

No synchronizing events need be available for the application of correlation techniques. The principal requirements are that the functions be stationary (i.e., their statistics do not change over the period of integration) and that the duration of each signal be sufficiently long in comparison to the period of its lowest frequency component.

Typical applications for correlation include:

- (1) Determination of the transmission paths and propagation velocities of acoustic waves, seismic waves and mechanical vibrations.
- (2) Measurement of the impulse-response characteristics of industrial processes and servo control systems in the presence of noise.
- (3) Indication of epilepsy through comparison of electroencephalograms from the two halves of the brain.
- (4) Analysis of the tremor frequencies of patients afflicted with Parkinson's disease.

THEORY

Stated mathematically, the autocorrelation function is defined as

$$\phi_{ff}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T f(t) f(t+\tau) dt, \quad (1)$$

where τ is the independent time-delay variable, $2T$ is the period of integration, and $f(t)$ is the time function being studied.

Since this is an even function [i. e., $\phi_{ff}(\tau) = \phi_{ff}(-\tau)$], the sign of τ may be changed from positive to negative. Equation (1) can then be rewritten as

$$\phi_{ff}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T f(t) f(t-\tau) dt. \quad (2)$$

This change in sign is necessitated by practical considerations, namely the fact that it is easier to delay a signal than to advance (or predict) it in time.

Crosscorrelation is defined as

$$\phi_{fg}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T f(t) g(t+\tau) dt, \quad (3)$$

where $f(t)$ and $g(t)$ are the two different time functions being studied.

Since

$\phi_{fg}(\tau) = \phi_{gf}(-\tau)$, this equation may be written as

$$\phi_{gf}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T f(t) g(t-\tau) dt. \quad (4)$$

A close approximation of the theoretical value of the auto- or crosscorrelation function can be achieved by digital computing techniques. Since any practical experiment must be conducted within a finite period, the closeness of this approximation is dependent upon the length of record T . In practice, for stationary functions, this record length should be at least one order of magnitude greater than the period of the lowest frequency component of interest.

In order to apply digital computing methods, the maximum time delay $\tau_{\max}$ is divided into N intervals of $\Delta\tau$ each, and the integral approximated through the summation of discrete partial products for each value of $n\Delta\tau$.

Rewritten in discrete form, equation (2) can be approximated as

$$\phi_{ff}(n\Delta\tau) \approx \frac{1}{k+1} \sum_{i=0}^k f(t_i) f(t_i - n\Delta\tau) \quad n = 0, 1, 2, 3 \dots N, \quad (5)$$

where $k = \frac{T}{\Delta t}$ is the number of sample intervals.

Similarly, equation (4) can be approximated by

$$\phi_{gf}(n\Delta\tau) \approx \frac{1}{k+1} \sum_{i=0}^k f(t_i) g(t_i - n\Delta\tau) \quad n = 0, 1, 2, 3 \dots N. \quad (6)$$

POWER SPECTRAL DENSITY COMPUTATIONS

Correlation can also be used as a basis for computing power spectral density functions. According to the Wiener-Khinchin theorem, the cross-spectral density function is defined as

$$\Phi_{gf}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_{gf}(\tau) (\cos \omega\tau - j \sin \omega\tau) d\tau,$$

where $\Phi_{gf}(\omega)$ is the cross-spectral density function of time functions $f(t)$ and $g(t)$, ϕ_{gf} is the crosscorrelation function of the two time functions, and the radian frequency ω is the independent variable.

Similarly, the power spectral density function is defined by

$$\Phi_{ff}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_{ff}(\tau) \cos \omega\tau d\tau.$$

These two equations provide a convenient means for analyzing random data on the basis of the distribution of their frequencies, as opposed to time correlations. This type of analysis will prove particularly effective in revealing frequency response characteristics of systems subjected to random excitation.

DESIGN CONCEPT

The Model 257 and Model 258 Correlators are special-purpose computers which employ both analog and digital techniques to compute the auto- and crosscorrelation functions of electrical signals. Both models are designed to operate on a real-time basis with Technical Measurement Corporation's Computer of Average Transients; the Model 257 is used with 400-address CAT^R instruments and the Model 258 with the 1024-address CAT^R 1000.

Unlike conventional correlation computers which calculate a single point of the correlation function for one complete set of data points, the Model 257 simultaneously

evaluates up to 256 correlation points while the input signal is present. Computed information is stored in digital form in the memory of the CAT. This information may be observed on the CAT's built-in cathode ray tube or read out through a variety of analog and digital recorders.

PRINCIPLE OF OPERATION

A simplified block diagram of the CAT/Correlator system is shown in Figure 1.

The computation technique employed by the Correlator is basically similar for both autocorrelation and crosscorrelation. One input is sampled periodically, converted into a digital binary number, and then stored in the memory of the Correlator. The time between samples Δt is made equal to the time delay increment $\Delta \tau$ for which the correlation function is to be computed. The sampling rate may be selected by means of a front-panel switch to obtain the maximum number of correlation points consistent with an adequate sampling rate for the frequency components in the input signal. (Because of the Correlator's on-line operation, a compromise must be made between the number of correlation points computed and the delay increment. Normally, four samples or more per cycle of the highest frequency component is acceptable.)

Each sample is converted into an equivalent digital value by means of a voltage-controlled oscillator which generates a pulse train whose frequency is proportional to the amplitude of the sampled waveform. Pulses from this oscillator are counted for a fixed period of time and stored in the Correlator memory with an additional bit which indicates the polarity of the sample.

When the memory of the Correlator is full, it contains the last 32, 64, 128, or 256 (as selected by a front panel control) sampled values of the first input function $g(t)$. For example, when 32 samples are taken (see Figure 2), the time delay $n\Delta \tau$ is created by sampling $g(t)$ at intervals of $\Delta \tau$ seconds apart, so that the last sample stored in the Correlator memory has a delay of 0, the next to last sample has a delay of $\Delta \tau$, etc. for all incremental values of n up to 31. After each new data sample, the entire contents of the memory are read out, and each number is multiplied by the real-time value $f(t_i)$ of the second input. Because of the speed of the multiplication process, this real-time value $f(t_i)$ is essentially constant for each series of partial products.

The digital values read out from the Correlator memory are converted into analog form by means of a current-summing ladder network and amplifier, then fed to one input of an analog multiplier. At the same time, another signal representing the real-time value of $f(t)$ is fed to a second input of the multiplier. This analog multiplier operates on the Hall effect principle. The first input generates a magnetic field while the second generates a current. This current is passed through a semiconductor wafer which is placed in a plane perpendicular to the magnetic field, producing a resultant voltage which is proportional to the product of the two inputs.

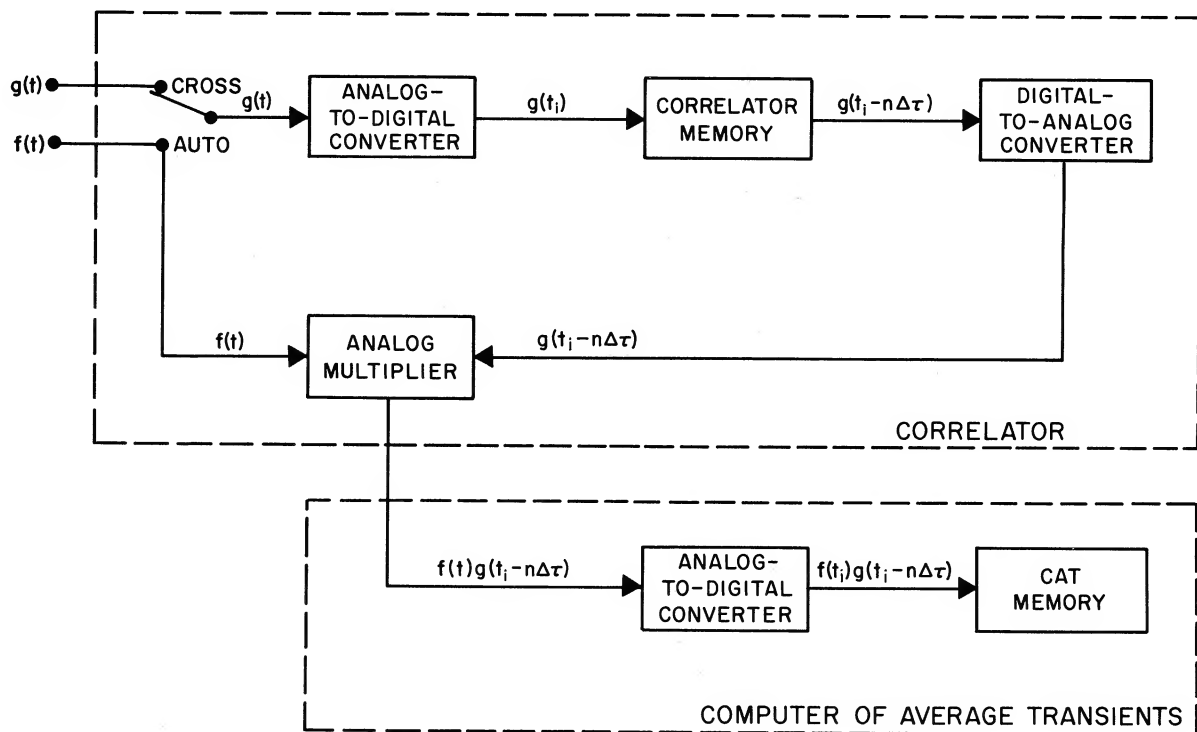


Figure 1. Simplified Block Diagram of TMC Correlation Computer System (input selector switch shown in position for crosscorrelation).

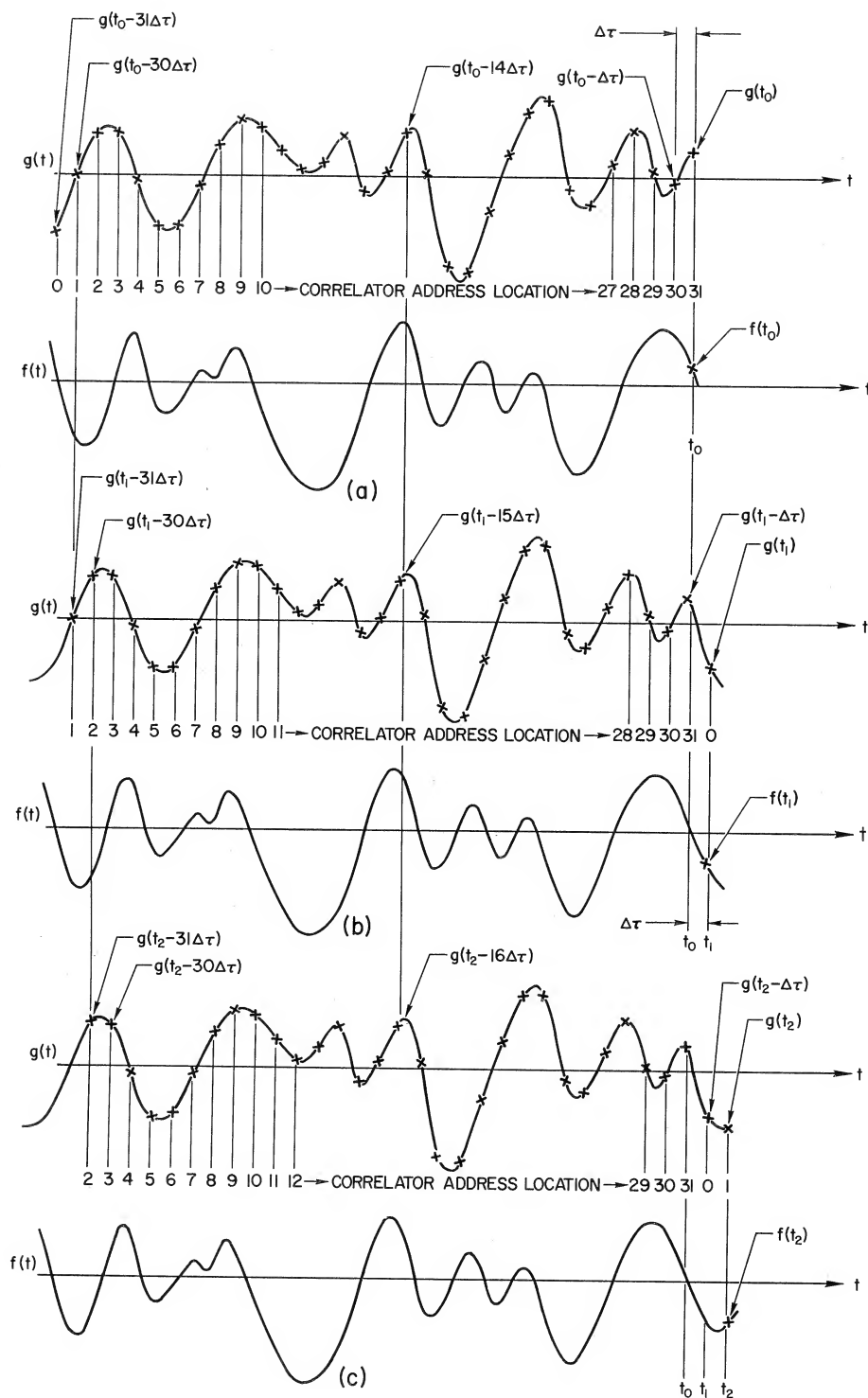


Figure 2. Correlator Memory Updating Process.

- (a) When the Correlator memory is filled at time t_0 , it contains 32 sampled values of $g(t)$, spaced at equal intervals of $\Delta\tau$ seconds apart. Within the next $\Delta\tau$ seconds, the entire contents of the memory are read out and multiplied by the second input $f(t)$.
- (b) After one readout cycle, the oldest sample $g(t_0 - 31\Delta\tau)$ is replaced by $g(t_1)$, and the readout and multiplication cycles repeated.
- (c) This same sequence is again repeated at time t_2 , with sample $g(t_2)$ replacing $g(t_1 - 31\Delta\tau)$ and a new set of partial products obtained.

In crosscorrelation computations, each series of partial products constitutes a set of values for $f(t_i) g(t_i - n \Delta \tau)$ for a particular value of t_i and all values of n . In autocorrelation computations, the process is similar except that two inputs are derived from the same source and the series of partial products represents a set of values for $f(t_i) f(t_i - n \Delta \tau)$ for constant t_i .

As each multiplication is performed, the partial product is read out to the memory of the CAT. Data is then stored in a series of successive CAT memory addresses, each of which corresponds to a specific value of n (see Table 1). For each sample, a new partial product is added to the data already accumulated in each memory address, so that the information contained in each address represents a cumulative total of $f(t_i) g(t_i - n \Delta \tau)$ for all values of t_i and for one particular value of n .

TABLE 1

REAL TIME	CAT ADDRESS NUMBER			
	0	1	2	n
t_0	$f(t_0) g(t_0)$	$f(t_0) g(t_0 - \Delta \tau)$	$f(t_0) g(t_0 - 2 \Delta \tau)$	$f(t_0) g(t_0 - n \Delta \tau)$
t_1	$f(t_1) g(t_1)$	$f(t_1) g(t_1 - \Delta \tau)$	$f(t_1) g(t_1 - 2 \Delta \tau)$	$f(t_1) g(t_1 - n \Delta \tau)$
t_2	$f(t_2) g(t_2)$	$f(t_2) g(t_2 - \Delta \tau)$	$f(t_2) g(t_2 - 2 \Delta \tau)$	$f(t_2) g(t_2 - n \Delta \tau)$
....				
t_k	$f(t_k) g(t_k)$	$f(t_k) g(t_k - \Delta \tau)$	$f(t_k) g(t_k - 2 \Delta \tau)$	$f(t_k) g(t_k - n \Delta \tau)$
	$\sum_{i=0}^k f(t_i) g(t_i)$	$\sum_{i=0}^k f(t_i) g(t_i - \Delta \tau)$	$\sum_{i=0}^k f(t_i) g(t_i - 2 \Delta \tau)$	$\sum_{i=0}^k f(t_i) g(t_i - n \Delta \tau)$

When the last partial product for each value of t_i is read into the CAT, the memory of the Correlator is automatically updated by replacing the oldest sample of $g(t)$ with the real-time value of the function. This value is immediately read out, multiplied by itself, and the product stored in the CAT address corresponding to zero delay. As this sequence is repeated, a continuous precession occurs so that the Correlator memory always contains the 32, 64, 128 or 256 most recent samples of $g(t)$.

The memory updating process can be visualized as continually storing data in discrete steps on a closed loop. At each revolution of the loop, the oldest data point is replaced by more current information.

A five-position switch is provided on the front panel of the Correlator to permit selection of a preset integration time T corresponding to 25, 50, 100, 200 or ∞ times the total time delay $\tau_{\max}$. This preset integration time control permits direct comparison of correlograms computed for the same integration time, provided that the CAT is operated at a constant analysis time.

In addition to the preset integration time control, automatic computation of a normalization factor is also provided by the CAT/Correlator system. This normalization factor, which is directly proportional to the integration time T , is stored in the CAT address immediately following the address containing the value of the correlation function for $\tau_{\max}$. The normalization count is accumulated during the period of time when the Correlator memory is being updated, and corresponds to a zero volt input to the CAT. Therefore, in addition to providing a measure of integration time, it also serves as a zero-volt-square reference point for the correlation function.* The most significant digits of the normalization factor are indicated by a Sodeco counter mounted on the front panel of the Correlator.

Also mounted on the Correlator's front panel are two controls for selecting the delay increment $\Delta\tau$. One switch selects both the number of correlation points and a corresponding delay increment. The second allows the selected delay increment to be multiplied by a factor from 1 to 32. The two switches provide a total range of delay increments from 2.5 to 640 milliseconds for the Model 257 and from 1 to 256 milliseconds for the Model 258.

PROPERTIES AND EXAMPLES OF AUTOCORRELATION

The autocorrelation function can be shown to have the following properties:

1. As noted earlier, the autocorrelation function is an even function, i.e., $\phi_{ff}(\tau) = \phi_{ff}(-\tau)$. Consequently, the function can be completely defined by calculating $\phi_{ff}(\tau)$ for positive values of τ only.
2. The value of the autocorrelation function for $\tau = 0$ is the mean square voltage of the time function. Substituting $\tau = 0$ in equation (2),

$$\phi_{ff}(0) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T f^2(t) dt = \overline{f^2(t)}.$$

3. The autocorrelation function has its maximum value for $\tau = 0$.
4. If the time function contains periodic components, the autocorrelation function will contain components having the same periods.

* The correlation function has the dimension of volts squared.

5. If the time function contains only random components (noise) and no periodic components, the autocorrelation function will approach zero as τ increases. The rate at which it approaches zero is a measure of the randomness of the time function and therefore of the bandwidth of the noise.
6. If the time function is composed of two or more independent components, the resultant autocorrelation function will be the sum of the autocorrelation functions of each individual component.
7. Each time function has only one possible autocorrelation function, but every autocorrelation function may be represented by an infinite number of time functions. This irreversible relationship arises from the loss of phase information when calculating the autocorrelation function.

Examples of five typical time functions and their autocorrelograms are shown in Figures 3 through 7 in order to establish a basis for interpreting the results of autocorrelation.

Figure 3(a) shows a simple sine wave whose time function is $f(t) = A \sin(\omega t + \alpha)$. Its autocorrelation function [Figure 3(b)] illustrates properties 2, 3, 4 and 7. The equation for this autocorrelation function is

$$\phi_{ff}(\tau) = \frac{A^2}{2} \cos \omega \tau.$$

For $\tau = 0$, the value of the autocorrelation function is $\frac{A^2}{2}$, the mean square value of the time function. Notice that the autocorrelation function has the same frequency as the time function and that the phase angle α is lost in the transformation.

Figure 4(a) shows a square wave whose time function can be expressed as a Fourier series

$$f(\omega t) = \frac{4V}{\pi} (\cos \omega t - 1/3 \cos 3 \omega t + 1/5 \cos 5 \omega t - \dots).$$

The autocorrelation function illustrated in Figure 4(b) is given by the equation

$$\phi_{ff}(\omega \tau) = \frac{8V^2}{\pi^2} (\cos \omega \tau + \frac{1}{3^2} \cos 3 \omega \tau + \frac{1}{5^2} \cos 5 \omega \tau + \dots).$$

This example illustrates properties 2, 3, 4 and 6. It can easily be shown that

$$\phi_{ff}(0) = \frac{8V^2}{\pi^2} (1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots) = V^2,$$

where V^2 is the mean square value of $f(\omega t)$. Both the time function and the autocorrelation function contain an infinite number of components, the fundamental and harmonic frequencies being equal in both.

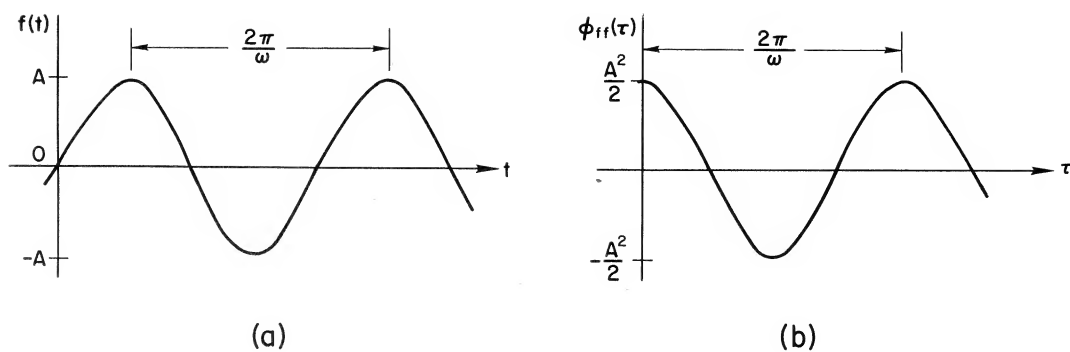


Figure 3. Time function (a) and Autocorrelation Function (b) of a Sine Wave Input.

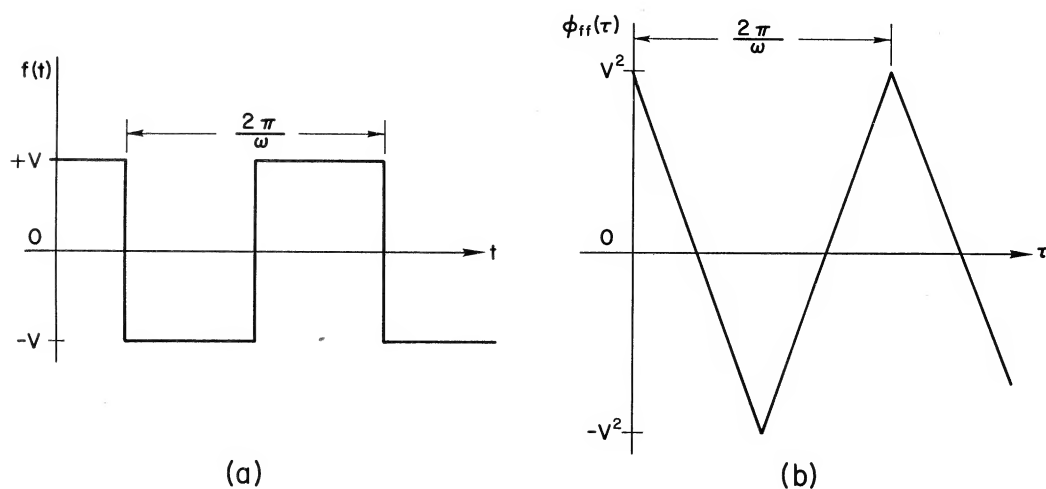


Figure 4. Time Function (a) and Autocorrelation Function (b) of a Square Wave Input.

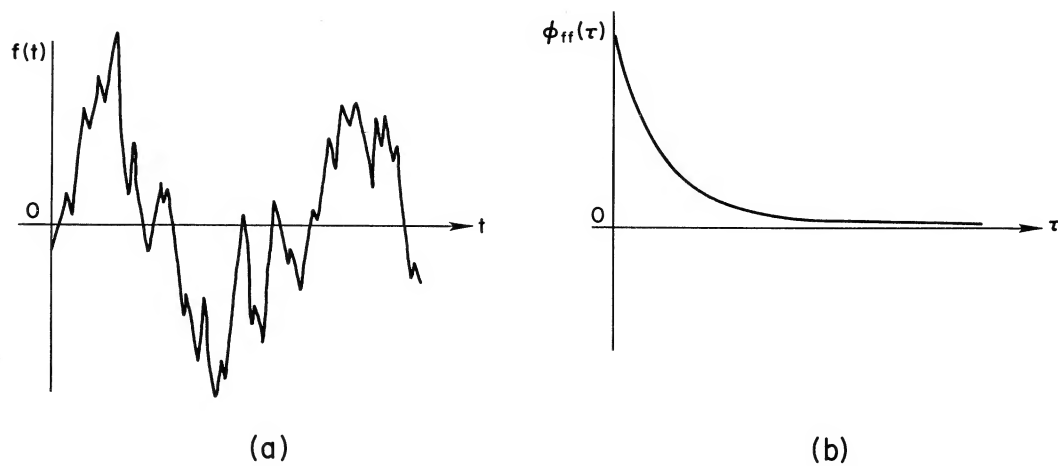


Figure 5. Time Function (a) and Autocorrelation Function (b) of a Wide Band Noise Input.

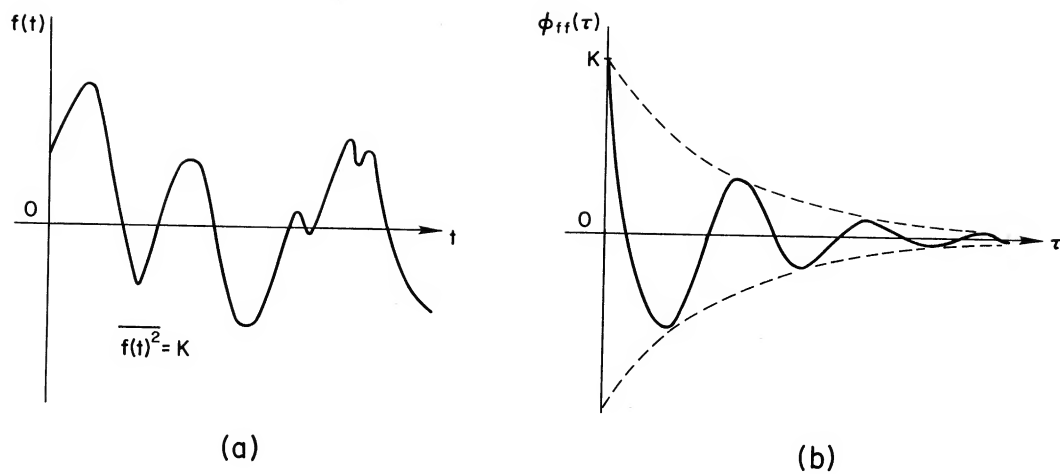


Figure 6. Time Function (a) and Autocorrelation Function (b) of a Narrow Band Noise Input.

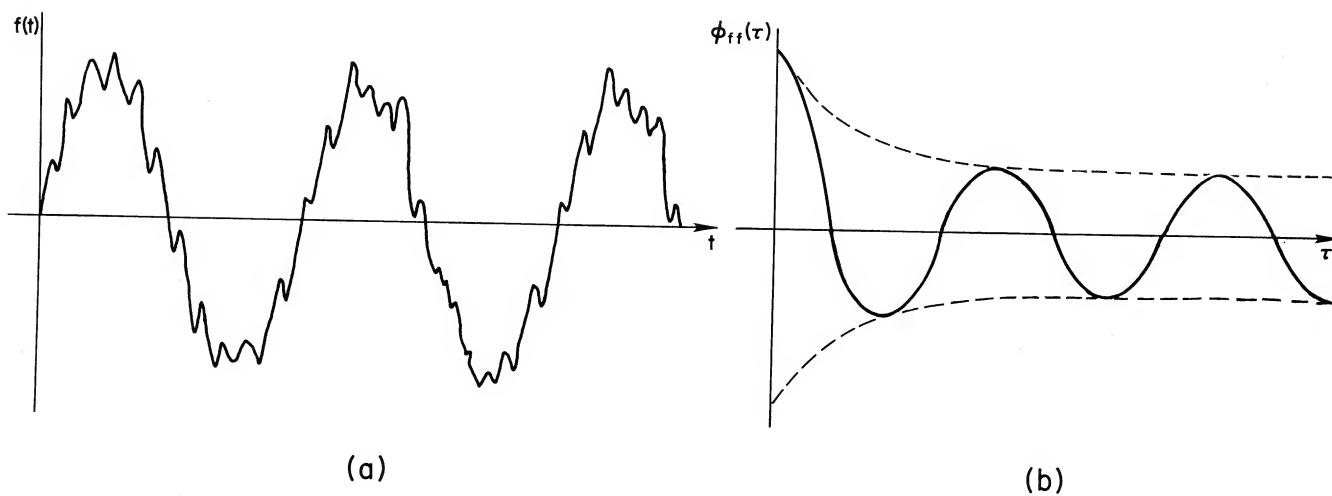


Figure 7. Time Function (a) and Autocorrelation Function (b) of a Sine Wave Input Containing Wide Band Noise.

Figure 5(a) represents a time plot of wide band noise (i.e., noise whose frequency spectrum is broad but finite). The autocorrelation function shown in Figure 5(b) has the form

$$\phi_{ff}(\tau) = Ke^{-\alpha\tau},$$

where K is the mean square value of the noise and α is a measure of its band width. The exponential decay of the autocorrelation function illustrates property 5.

Figure 6(a) is a time plot of narrow band noise (i.e., noise whose frequency spectrum is narrow relative to its center frequency). The autocorrelation function in Figure 6(b) has the form

$$\phi_{ff}(\tau) = Ke^{-\alpha\tau} \cos \beta \tau,$$

where K is the mean square value of the noise, α is a measure of its band width and β is a measure of its center frequency. As in Figure 5(b), the autocorrelation function tends toward zero as τ increases.

Figures 7(a) and 7(b) show the time plot and autocorrelogram, respectively, of a sine wave with wide band noise superimposed. The correlogram, which is a composite of Figures 3(b) and 5(b), has the form

$$\phi_{ff}(\tau) = Ke^{-\alpha\tau} + \frac{A^2}{2} \cos \omega \tau.$$

This example illustrates the effectiveness of autocorrelation in detecting periodic components obscured by noise. Since the amplitude of the autocorrelation function at $\tau = 0$ will be $\frac{A^2}{2} + K$ and its envelope is asymptotic to $\frac{A^2}{2}$, autocorrelation provides a convenient means for determining the rms signal-to-noise ratio $\sqrt{\frac{A^2}{2K}}$ of the composite time function.

PROPERTIES AND EXAMPLES OF CROSSCORRELATION

The three basic properties of the crosscorrelation function are:

1. The crosscorrelation function $\phi_{gf}(\tau)$ is not necessarily an even function, and therefore in general $\phi_{gf}(\tau) \neq \phi_{gf}(-\tau)$. Consequently, a time shift of one input function in one direction with respect to the other does not produce the same result as an equal shift of the same function in the opposite direction.
2. $\phi_{fg}(\tau) = \phi_{gf}(-\tau)$. A shift of one input function $f(t)$ in one direction must yield the same result as an equal shift of the other input function $g(t)$ in the opposite direction.

3. $\phi_{gf}(\tau)$ does not necessarily have a maximum at $\tau = 0$. The maximum value of $\phi_{gf}(\tau)$ will occur for that value of time shift τ for which the two input functions are most alike.

To illustrate the application of crosscorrelation computation, four typical examples are given in Figures 8 through 11.

Figure 8(a) shows an input signal disturbed by random noise. In order to determine whether the signal contains a periodic component of frequency ω_0 , a sine wave of that frequency [Figure 8(b)] is generated and crosscorrelated with the input signal.

Let the time function of the unknown signal be

$$f(t) = S_1(t) + N(t),$$

where $S_1(t)$ represents the periodic component (a square wave) and $N(t)$ represents the noise component, and let the time function of the reference signal be

$$g(t) = B \sin(\omega_0 t + \beta) = S_2(t).$$

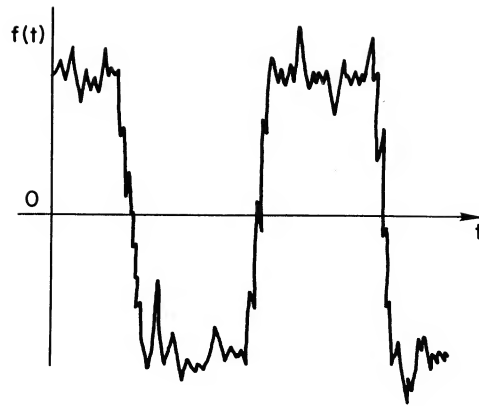
The crosscorrelation function may then be written as

$$\begin{aligned} \phi_{gf}(\tau) &= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T [S_1(t) + N(t)] S_2(t - \tau) dt \\ &= \phi_{S_2 S_1}(\tau) + \phi_{S_2 N}(\tau). \end{aligned}$$

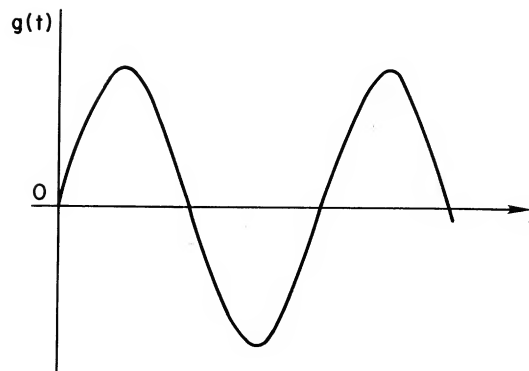
This yields a result which is the sum of two crosscorrelation terms, one of the unknown signal with the reference signal, and the other of the noise with the reference signal. Assuming that $N(t)$ is statistically independent of $S_2(t)$, the term $\phi_{S_2 N}(\tau)$ will equal zero. The remaining term is a sinusoidal function [shown in Figure 8(c)] whose frequency is common to $f(t)$ and $(g)t$, and whose phase angle is equal to the difference between the phase angles of the common component in the two time functions.

For a given record length, this method of signal-in-noise detection will provide a greater improvement in signal-to-noise ratio than can be obtained through autocorrelation techniques. This advantage can be seen by comparing the number of noise terms appearing in the cross- and autocorrelation functions, the crosscorrelation function having only one while the autocorrelation function contains three.

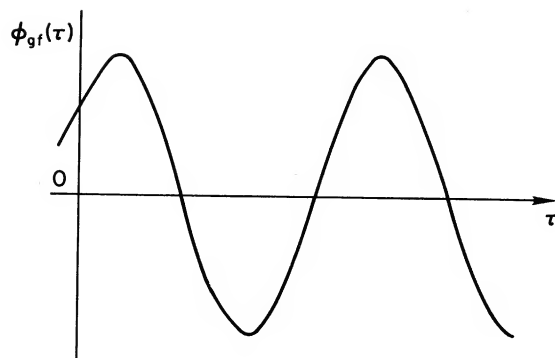
Another method for detecting a signal obscured by noise is illustrated in Figure 9. This method consists of crosscorrelating the obscured signal [Figure 9(a)] with a series of narrow pulses [Figure 9(b)] which has the same fundamental frequency as the obscured signal. It can be shown that the resultant crosscorrelation function, shown in Figure 9(c), has a waveshape identical to that of the hidden signal.



(a)

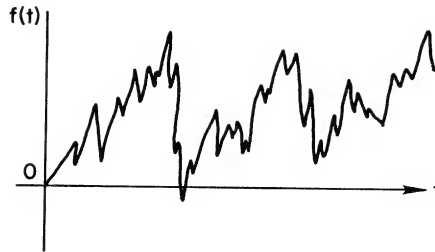


(b)

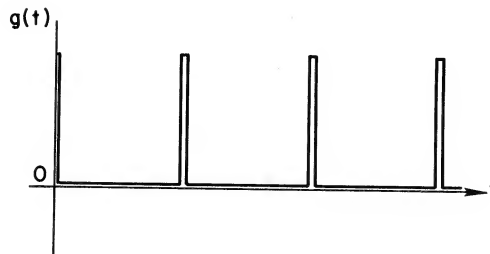


(c)

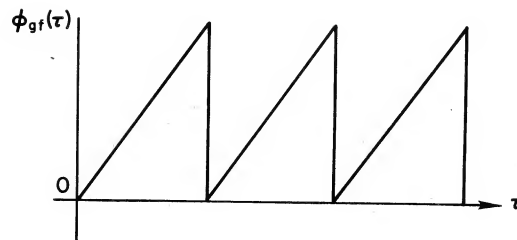
Figure 8. (a) Time Function of a Square Wave Input Containing Random Noise, (b) Time Function of a Sine Wave Generated at Same Frequency as the Square Wave Input, and (c) Crosscorrelogram Resulting from Crosscorrelating Functions Shown in (a) and (b).



(a)



(b)



(c)

Figure 9. (a) Time Function of a Triangular Wave Input Containing Random Noise, (b) Time Function of a Series of Narrow Pulses Having the Same Fundamental Frequency as the Triangular Wave, and (c) Crosscorrelogram Resulting from Crosscorrelating Functions Shown in (a) and (b).

The obvious advantage of this method over autocorrelation is that of preserving phase information. Consequently, the resulting crosscorrelogram is a proportional replica of the original time function.

This method also offers two advantages over the technique illustrated in Figure 7. First, it will identify the complete signal waveform as opposed to a single frequency component; and second, for a given record length, it will produce an even greater improvement in signal-to-noise ratio.

Illustrated in Figure 10(a) is a possible acoustics application. The problem is to determine the contribution made by noise source G to the overall noise level at point F.

A microphone placed at point F will detect the overall noise level

$$f(t) = A(t - \alpha) + B(t - \beta) + C(t - \gamma) + \dots + G(t - \theta).$$

A second microphone is so located as to detect only the sound produced by source G

$$g(t) = G(t).$$

Crosscorrelating $f(t)$ and $g(t)$,

$$\phi_{gf}(\tau) = \phi_{GA}(\tau) + \phi_{GB}(\tau) + \phi_{GC}(\tau) + \dots + \phi_{GG}(\tau).$$

Assuming that the sound produced by each noise source is statistically independent of that produced by all other sources, the terms ϕ_{GA} , ϕ_{GB} , ϕ_{GC} , ϕ_{GD} and ϕ_{GE} will each equal zero. A plot of the resulting crosscorrelation function is shown in Figure 10(b). The peak value of the function occurs for $\tau = \theta$, where θ is transit time from point G to point F.

If multiple sound paths exist, the correlogram will contain one peak for each path. The value of the abscissa at each peak will correspond to the transit time for the related sound path, while the ordinate of each peak indicates its relative contribution.

If the second microphone is moved to points A, B, C, D and E in sequence and a similar crosscorrelation computation performed for each point, the individual correlograms obtained will provide a basis for determining the contribution of each source to the overall sound level at point F.

The use of crosscorrelation in measuring system impulse-response characteristics is illustrated in Figure 11. Function $f_d(t)$ represents an uncontrollable and unmeasurable disturbance acting upon the system; $f_i(t)$ and $f_o(t)$ are the system input and output functions, respectively. The problem is to determine $h(\tau)$, the unit impulse-response of the system.

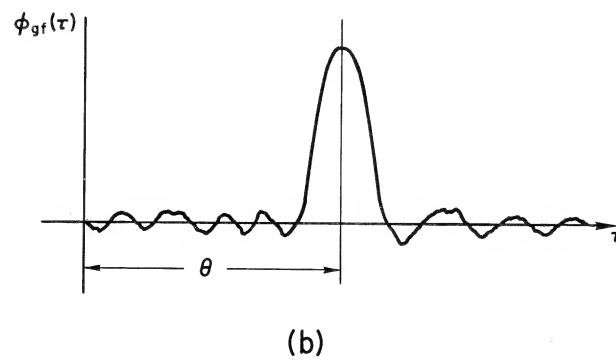
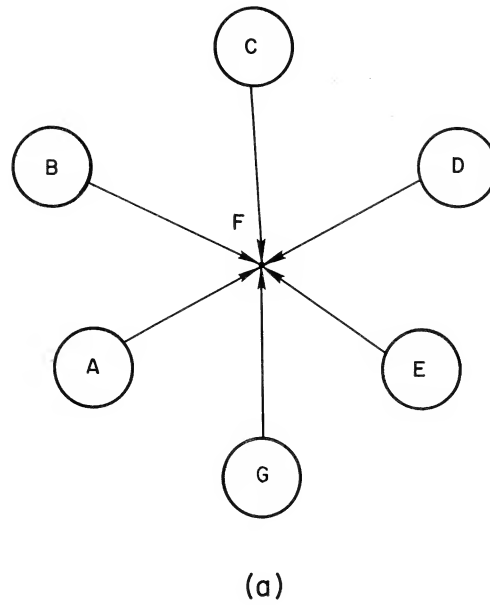


Figure 10. (a) Location of Multiple Noise Sources with Respect to Point F and (b) Correlogram Resulting from Crosscorrelation of Output of a Microphone Placed at Point G with Output of a Second Microphone at Point F .

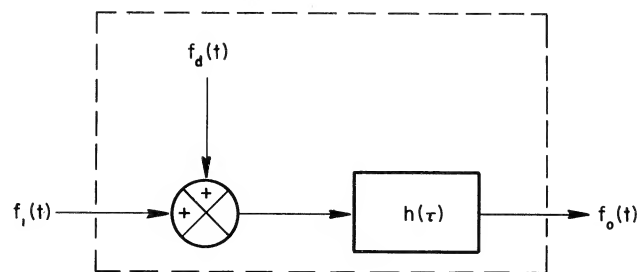


Figure 11.

A direct measurement of $h(\tau)$ cannot be made by applying a unit impulse to the input of the system because the output would contain a contribution due to $f_d(t)$, the system disturbance. However, it can be shown that the crosscorrelation function of the system input $f_i(t)$ with its output $f_o(t)$ in terms of the impulse-response function $h(\tau)$ is

$$\phi_{io}(\tau) = \int_{-\infty}^{\infty} h(\lambda) \phi_{ii}(\tau - \lambda) d\lambda + C,$$

where C is a constant.

This equation is the convolution of the system impulse-response function with the input autocorrelation function. The constant C can be made equal to zero if the time average of either $f_i(t)$ or $f_d(t)$ is zero. Therefore, if one were to supply an input consisting entirely of wide band random noise (whose autocorrelation function is itself a unit impulse), the resultant crosscorrelation of input with output yields the system impulse-response $h(\tau)$.

Although necessarily incomplete, these examples serve to indicate how powerful a tool correlation can be for analyzing otherwise indeterminate signals and their effects.

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